Mathematics Intervention in the Primary Grades

What are effective intervention strategies for students struggling in mathematics?

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Research Question

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Rationale

Classroom experience and personal interest have encouraged me to think deeply about children and mathematics. I have been teaching for ten years. During this time, I have done much professional development in the area of mathematics. I have served in the role as a math teacher leader for seven years and have mentored less experienced teachers in the area of mathematics. Two years ago, I designed the curriculum for an after school program for students at-risk in mathematics, which provided the opportunity for me to see struggling students across grade levels. My experience suggested to me that despite my best efforts, certain students were not grasping grade level material. These children struggle with mathematics and, it seemed to me that normal classroom supports, including differentiated instruction, seem to have little impact on their progress. Hence, for this research paper, I decided to focus on the students who were struggling with math in my fourth grade class over a two-year period.

PS 41, The Greenwich Village School, is considered a high performing school and is exempt from the Chancellor's mandatory curriculum. Seven hundred thirty-five students attend the school. The ethnicity of the population is as follows: 70% of the children are White, 14% are Hispanic, 6% are Black, and 10% are Asian and other ethnicity children. Nine percent of the children are eligible for free lunch. In 2005-2006, ninety-five percent of all students met or exceeded performance standards on the statewide test. Fifteen students did not meet standards.

In addition to these fifteen students, students who receive a low three on the test often struggle with grade level material in the classroom. During my two years research I identified, as well, two students each year who scored a mid-level three on the test but were considered at risk for succeeding in mathematics based on teacher assessments.

In 2003 the stakes were raised in mathematics when the new mayor announced the end of social promotion. In the past, the promotion criteria were based on a child's reading level; now, with the mayor's mandate, promotion criteria were expanded to include both reading and mathematics. While the prior emphasis on literacy had enabled the development of a good support system to help students struggling with reading, there were no similar structures for children struggling with mathematics. This disadvantage was compounded by the fact that many elementary school teachers feel a stronger affinity and competence for teaching literacy over than mathematics. As a math teacher leader, I found that those teachers who were focused on supporting at-risk students found that simply differentiating the daily curriculum was inadequate to support these children.

As I thought about this problem I looked to the Reading Recovery Program for ideas. Reading Recovery is a successful intervention program designed by Marie Clay (1972) as an intervention for children struggling in reading. The Reading Recovery Program is based upon a particular philosophy of teaching and learning. Unlike phonic approaches to reading that stress decoding, Reading Recovery comes out of the whole language community in New Zealand and emphasizes reading for meaning. The Ministry of Education in New Zealand (1995) describes reading as "…an interaction between what is on the page and what is in the reader's head…Skilled reading, therefore, is not a process of identifying all the letters, and linking these to make words which in turn make sentences conveying meaning" (p. 25).

The Reading Recovery has built its success on identifying children early (usually in 1st and 2nd grade) and working with them on a one to one basis. It has a proven track record of success. I began to ask if there was something we could learn from this model that might transfer to how we work with students struggling in mathematics. In addition to looking at the Reading Recovery model, in February of 2006 the NYC Department of Education mandated that teachers work an additional period a day with struggling students in an Extended Day Program. The Extended Day program holds promise because it provides an opportunity to work with students in small groups. The question is how best to use this time with students struggling in mathematics.

Review of the Literature

For years, controversy raged in the literacy world between the advocates of a phonics approach to teaching reading versus the whole language method. Although the debate has not been completely extinguished, there are now many reading programs that blend the two approaches into what is called a Balanced Literacy Program (New York City, 2003). There is a similar controversy in the mathematics world. People on one side of the debate emphasize learning facts and using traditional algorithms to solve problems. People on the other side advocate an approach that places emphasis on students taking an active role in constructing knowledge and developing what is called "number sense".

Number sense in mathematics is similar to reading for meaning in literacy. A child may be able to decode what is written on the page but if they are not reading for meaning, there is little comprehension. In mathematics, it may appear that a child is able to add and subtract with an algorithm, but once the numbers become larger and borrowing and carrying are involved the flaw in a child's understanding is exposed. One can see that the child has not developed a sense of the relationships between numbers and how numbers can be composed and decomposed. The child's thinking is reflected in the tasks; when asked to work with numbers that involve a more sophisticated sense of place value the child's understanding breaks down and mistakes are made.

A student with number sense has knowledge about how numbers are structured in our number system. They have a large repertoire of known number relationships and a mental model of what it means to add, subtract, multiply and divide. With the right experiences, children begin to notice the patterns that reveal the structure of our number system. An example given in the first grade TERC curriculum unit called *Building Number Sense* (1995) is that children notice that the thirties come after the twenties, the forties after the thirties and so forth, and they come to see that the sequence within each decade has common features. Thus, number sense is a key concept that must be mastered by children early in their learning about mathematics.

The concept of number sense is based on a constructivist view of teaching and learning. Constructivist thought draws from the work of Piaget, a psychologist who studied the development of children's thinking. According to Labinowicz (1980), "Piaget noticed patterns in children's responses to intellectual tasks which he interpreted as reflections of children's level of reasoning. Children of like age groups responded in ways that were remarkably similar to each other and remarkably different from adult responses and expectations." (p. 21)

In contrast to those who view the child as a passive vessel into which knowledge is poured, Piaget determined the child to be an active participant in the construction of knowledge. According to Piaget, a child constructs knowledge through the processes of assimilation and accommodation. As Labinowicz (1980) describe these processes, assimilation occurs when a child encounters new ideas that fit into his current understanding of how the world works and is able to incorporate these ideas into his current mental structures thereby expanding his worldview. Accommodation occurs when the child encounters something that contradicts what he currently believes to be true about how the world works. When this occurs, a state of disequilibrium sets in. The mental tension produced by the disequilibrium provides the motivation for the child to do the intellectual work necessary to reconcile what he thought to be true with the new information he has received. As the child adapts his current worldview, by changing an inaccurate idea into one more in line with the reality he has discovered, he uses the process of accommodation. Through using both assimilation and accommodation the child is constructing knowledge. Implicit to Piaget's work is a developmental theory that suggests that certain fundamental tasks need to be mastered before more complex ideas can be integrated into an individual's intellectual framework.

People in many fields, including education, have built upon Piaget's work. Fosnot and Dolk (2001) have been instrumental in researching and documenting how children's mathematical thinking unfolds. Like Piaget, they view learning mathematics as a process in which children are making discoveries and actively constructing knowledge in a developmental fashion. They believe that traditionally mathematics has been taught as a "dead language", that is, what mathematicians have discovered in the past is presented to students as something to be learned, practiced and applied (p. 12).

Fosnot and Dolk (2001) have mapped out a *learning landscape* that outlines the mathematical development of children's ideas. This landscape contains markers that reflect children's thinking as their development unfolds. A teacher trained to teach mathematics in this way is able to identify these markers through specific tasks that a child completes. The teacher is not only fluent in mathematical content knowledge but also in the intellectual development of a child as it applies to mathematical knowledge. A teacher familiar with the learning landscape is able to identify important mathematical ideas and strategies that a child grasps, which reflect important landmarks in the children's understanding of mathematics (p. 9).

Central to understanding the learning landscape are what Fosnot and Dolk (2001) refer to as *big ideas*:

> Big ideas are the central, organizing ideas of mathematics – principles that define mathematical order. As such, they are deeply connected to the structures of mathematics. They are, however, also characteristic of shifts in learners' reasoning – shifts in perspective, in logic, in the mathematical relationships they set up…These ideas are "big" because they are critical to mathematics and because they are big leaps in the development of children's reasoning. (p. 10).

As an example of a *big idea*, Fosnot and Dolk describe *unitizing*. Unitizing occurs when a child is able to grasp that a single object may stand for more than one thing, for example, a nickel is equal to five pennies. To understand this concept, children must be able to hold onto the idea that a number is useful not only for counting objects but also for representing a group.

If a child grasps such big ideas at the time appropriate to his age level, he is able to stay abreast of the curriculum. As the child moves from grade to grade, certain early understandings are taken for granted. It is thought that struggling students may not have mastered the developmental tasks, and the big ideas that are reflected through those tasks, during the early stages of their mathematical development. Thus, they are handicapped when the curriculum moves forward. Often these children rely heavily on an algorithm as a way of compensating for what they do not understand – if they are able to memorize the steps in a procedure it gives them a way to access a problem. Unfortunately, if they forget the steps or are presented with an open ended mathematical situation, they are not able to reason their way through because they do not truly understand how the number system works. What these children need is not the slower pace of differentiation but a focused intervention like Reading Recovery that enables them to recapture those concepts that they missed.

If all children are to be mathematically successful, in addition to an intervention strategy like Reading Recovery, quality professional development needs to be available to teachers. This will allow the instruction and differentiation that teacher's provide in the classroom to be optimally effective. Fundamental to the idea of children progressing along the learning landscape is a teacher who is familiar with the mathematical development of children's ideas as well as fluent in mathematical content knowledge. It is well documented that there is a correlation between teacher learning in the area of mathematics and student learning (Cohen and Hill, 1998; Stigler and Hiebert, 1999; Hill and Ball, 2004).

In Cohen and Hill's (1998) extensive study of over 600 elementary school teachers in California, the researchers found that student achievement in mathematics was significantly related to high quality professional development for teachers. Cohen and Hill found that only a minority of teachers in their study were receiving would they considered high quality professional development, but in those cases where it did exist, it made a significant difference. According to Cohen and Hill most professional development done in the United States is fragmented and does not focus on the curriculum that the teacher is using in the classroom. Cohen and Hill concluded that to make a difference in student learning, professional development must be:

- Grounded in the curriculum that students study;

- Connected to several elements of instruction (for example not only curriculum but also assessment); and
- Extended in time (Cohen & Hill, 1998, p. 24).

Data Collection

I have used three tools to collect data to document student progress and growth of mathematical thinking: a standardized mathematics test and teacher assessments, student work collected over time, and notes based on observations and discussions with the students.

Standardized Mathematics Test and Teacher Assessments

In order to identify early in the year which of the students in my fourth grade class were struggling in mathematics, I looked at the results from the previous year's third grade standardized mathematics test. I should note that in 2006 the City administered the tests and in 2007 they were administered by the State. Although they were different tests, I found they generally correlated well with classroom performance and thus were a good base indicator for identifying students struggling in math. However, both years I had two students who were determined to be proficient in terms of the standardized tests but performed poorly on class assessments and thus were also included in the study. The standardized tests have four levels of performance defined as follows:

- *Level 4* – Students exceed the learning standards for mathematics. Their performance shows a superior understanding of key math ideas.
- *Level 3* – Students meet the learning standards. Their performance shows a thorough understanding of key math ideas.

- *Level 2* – Students show partial achievement of the standards. Their performance shows only partial understanding of key math ideas.
- *Level 1* – Students do not meet the standards. Their performance shows a minimal understanding of key math ideas.

Participants

During 2006 twelve students were included in my study and in 2007 eight students were included. Two students from each year scored a solid Level 3 but performed poorly on in-class teacher assessments.

Performance on Third Grade Standardized Math Test

	Total # of Students	Level 1	Level 2	Low Level 3	Mid Level 3
2006	12	1	7	2	2
2007	8	0	0	6	2

Student Work

Student work was analyzed at regular intervals over the year to document student progress and identify areas of continued difficulty. In the early part of the study, teacher assessments focused on a wide range of mathematical topics to identify the areas in most need of instruction. From the initial assessment it was determined that the areas of greatest need were mathematical operations and number sense. At that point, student instruction and work focused on achieving skills and understanding in these areas.

Field Notes

Throughout the course of the study I kept notes, which documented my observations, conversations, and interactions with children individually and in small groups when we played games together or engaged in other math-related activities. These notes helped me understand their mathematical thinking.

Intervention and Analysis

Using the results of the third grade standardized math test and fourth grade in class assessments, I identified 12 students out of a class of 29 who were struggling in mathematics during the 2006 school year and 8 students out of a class of 30 during 2007 school year. All students were developmentally far enough behind their peers in mathematics that differentiating the daily lesson did not address their needs. They were, in the terms of Fosnot and Dolk (2001), at different points on the learning landscape than their mainstream peers.

I worked with the struggling students during our Extended Day Program, which met Monday through Thursday from 8:00 to 8:35 am. The advantage of this type of small group intervention was that I could do lessons that were different than those covered in the grade level curriculum. Thus, I was able to target the specific needs of the children in the small group.

In 2006, seven students in the group of twelve had a basic understanding of the number system and in 2007 there were six students in the group of the eight. These students primarily needed support as the classroom-based material became more difficult, specifically, multi-step word problems, subtraction that involved borrowing and division. These are issues that are readily addressed by small group instruction. The students who needed this type of intervention seemed to benefit from the small group intervention and were performing on grade level within six to eight weeks. They no longer needed to attend the extended day program but I monitored their progress closely during the regular school day and would ask them to come to come in on an as needed basis. Once these students were discharged from the program I was able to work with an even smaller group of students in the weeks following.

During 2006, of the five remaining students, two had non-mathematics-related issues that were interfering with their performance: One student was having family problems that were interfering with his motivation and ability to concentrate. The other child was an English Language Learner who had come from Peru three years earlier and was still receiving ELL mandated services. Some progress was made but because it appeared that there might be learning issues unrelated to English language learning, he was referred for a Special Education evaluation at the end of the year.

In 2007 I had fewer students in the class who were identified as struggling in mathematics, 8 students out of a class of 30.   Two out of the eight students could not be brought up to grade level within an eight-week period but were not as significantly behind their peers as the students I worked with in 2006. In 2006, 12 out of 29 students needed intervention and within eight weeks, 7 out of the 12 students were discharged from the Extended Day Program; three students could not be brought up to grade level with the Extended Day Program alone. The three students in 2006 were much further behind their peers than the two most struggling students in 2007. This difference between cohorts seemed to be within the normal variation that one sees between classes from year to year. However, it highlighted the fact that over a two-year period, I identified five students who would have been better served by a program like Math Recovery as opposed to the Extended Day Program.

In 2006, the three students who were the furthest behind their peers shared similar characteristics regarding their mathematical thinking: They did not know the combinations of ten (7+3 =10) or how to make jumps of ten (22 +10 =32). They also could not easily sequence numbers through 100. These are all developmental tasks that reflect a child's level of thinking. These tasks involve more then mere memorization. If by fourth grade, a child has not mastered these tasks, the work of Fosnot and Dolk (2001) and others suggests that a fundamental developmental task has not been met. Such students cannot be brought up to grade level with short term, small group intervention. They need a type of mathematics intervention similar to the Reading Recovery program in its intensity. They need such an intervention because the ideas and activities that lay the foundation for understanding the number system at a fundamental level are not something that comes up for review or re-teaching in the grade level curriculum and they do not respond to short term, small group intervention. For an intervention to be effective, it is necessary for the child to go back and "recover" certain mathematical concepts that are crucial for developing number sense. These students need an intervention model that allows them to recoup or "recover" an understanding of critical concepts in mathematics, which are generally established by the end of second grade.

As a classroom teacher I am not in the position to offer an intervention like a Math Recovery Program because it requires working with one or two students on a daily basis for 12 – 14 weeks. However, 16 students over a two-year period did benefit from the intervention that I provided, which took place during the Extended Day Program. This intervention was grounded in constructivist mathematics. I analyzed the students' needs based on an understanding of the development of children's mathematical ideas. As I worked with children, I would analyze the types of errors made to determine the underlying concepts that were missing. For instance, subtraction when borrowing is involved is a common area of difficulty for struggling students. Even though, at the beginning of the year I do not teach the traditional algorithm students often come to fourth grade knowing this method having learned it at home or in Resource Room. An example of a problem that I use to analyze a student's understanding is:

$$\begin{array}{r} 1025 \\ \underline{-\ 647} \end{array}$$

When children need to borrow in the ones, tens and hundreds place errors are often made. When analyzing the errors made, it became apparent that the children often lacked a strong sense of place value. In this situation I would expand the numbers so that it becomes transparent to the children what they are "borrowing" from:

$$\begin{array}{rrrrr} & 1000 & +\ 0 & +\ 20 & +\ 5 \\ \hline - & & 600 & +\ 40 & +\ 7 \end{array}$$

By presenting it this way a student can see, for example, that they cannot subtract 7 from 5 so they must look to the tens place. As they move over a group of 10 to the ones place they are left with 20 in the tens place. Then as they try to subtract 40 from 20 they look at the hundreds place and see there are no hundreds to borrow from. They must look to the thousands place and move a group of 1000 over to the hundreds place. Now they can subtract 600 from 1000. As they do this calculation I have them write the actual number that they are moving over, such as writing 1000 above the hundreds place when they borrow. Then they actually cross out the 1000 and turn it into 900 when they move a group of 100 to the tens place to make the number 130. They will also write 130 in the tens place so that it becomes transparent that they have added 100 to they 30 that was there to get 130. This has the advantage over the traditional algorithm where they do not actually see the number but would see a small number 1 on top of the 0 in the hundreds place, for example.

$$\begin{array}{r} {\scriptstyle 9\;\;1\;\;1} \\ \cancel{1}\;\cancel{0}\;\cancel{2}\;5 \\ -\;\;\;6\;4\;7 \\ \hline 3\;7\;8 \end{array} \qquad \text{versus} \qquad \begin{array}{rcrcrcr} & & {\scriptstyle 900} & & & & \\ & & {\scriptstyle \cancel{1000}} & & {\scriptstyle 110} & & {\scriptstyle 1} \\ \cancel{1000} & + & \cancel{0} & + & \cancel{20} & + & 5 \\ - & & 600 & + & 40 & + & 7 \\ \hline & & 300 & + & 70 & + & 8 \end{array}$$

Struggling students sometimes cling the most determinedly to the algorithm. I have often tried to get a student to use another method, such as the number line, to deepen their understanding of the relationship between the numbers they are using. However, I noticed that they would use the method I encouraged when I worked with them but then when I looked at their independent work they had reverted back to the algorithm. I also discovered that children who go to Resource Room are taught the algorithm because pedagogically, the teacher considers it a good method for these learners. From my point of view, it did not seem to empower the students to deny the method they were using. My thinking was, if they were going to use the algorithm I wanted to strengthen their understanding of place value and why the algorithm worked, which would build their number sense in the process. In addition, when students enter fifth grade they are taught long division and embedded in this process is the traditional subtraction algorithm so it seems important that they know how to use it.

My interventions during Extended Day in the second year of my study focused primarily on the issue of place value as it came up it addition, subtraction, multiplication and division problems. In the first year of my study I had students that required me to go back even further in the progression of mathematical ideas to build their sense of number. Three of the students had difficulty taking jumps of ten (22, 32, 42) and sequencing numbers up to 100 (ask a student to begin counting from 53 to see if they can do it fluently). In this case as I analyzed the mistakes made I realized that I needed to work with the students on understanding the structure of the number system. I worked with students using a 100 chart until they could see the patterns in the number system. Once they were able to see the patterns in the 100 chart it made sense to them how to take jumps of ten and the way in which the decades (10, 20, 30, etc.) followed similar sequencing.

Conclusion

The results of the math intervention work that I did with children over a two year period has several implications for policy. First, I found that the needs of struggling students were best met in two very different ways - some benefited from short term, small group instruction such as what could be provided in an Extended Day Program while others required a more intensive intervention such as a Math Recovery model. Second, for students to benefit from an Extended Day model, a specific type of instruction is required; one that does not just shadow the daily curriculum but attempts to recover fundamental math concepts that were not mastered at an earlier stage. And third, for these interventions to be effective teachers need to receive high quality professional development as recommended by researchers such as Cohen and Hill (1998) Stigler and Hiebert (1999) and Ball (2005).

During my ten years of teaching at PS 41, which is within District 2 in Manhattan, I have received much high quality professional development. This professional development has demonstrated those characteristics that Cohen and Hill describe as being necessary if there is to be a significant correlation between teacher professional development and student achievement (Cohen & Hill, 1998). Thus, the results that I have achieved in increased student performance over the course of this research study are in line with what Cohen and Hill would predict.

Policy Recommendations

- Identify students in second grade who are a year behind grade level in mathematics. As a part of a Tier 2 Intervention, provide access to a Math Recovery Program. Teachers can be hired part time to provide this intervention. Teachers should meet between three and five days a week on a one to one basis with the student. Retired teachers and teachers on maternity leave who are strong in mathematics are good candidates for this position.
- Develop an Extended Day curriculum that can assess a student in terms of the development of mathematical ideas and then scaffolds learning to grade level material. A strong math teacher at each grade level can be paid per session to develop the curriculum.
- Provide high quality professional development for teachers, as defined by researchers, which shows a correlation between teacher learning and student achievement.

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