# Writing In Math

***Does Right = Write ?***

How Does Student Writing About Math Connect to the

Assessment of Students' Mathematical Understanding?

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## Research Question

How does student writing about math connect to the assessment of students' mathematical understanding?

## Rationale

As the Math Lead Teacher in Grade 3 during the school year 2004-2005, I was concerned about an apparent disconnect between math understanding and "getting the right answers" on work pages and quizzes.

Using the University of Chicago Everyday Math Program, I became aware that frequently students would complete activity pages, but were unable to explain what they had done to solve a problem. I was also very aware that "getting the answer" was all that mattered. The process seemed not important. And, I wondered after the answer was determined, did the students understand what they had done. The third grade city-wide testing in math is done in a multiple choice answer format. When the students reach the fourth grade the state math test contains a section, which is not multiple

choice, in which the students are required to show the steps taken in solving problems. This action research attempted to find out what the students truly had as a knowledge base as a way to develop mathematical confidence.

P.S. 64 on Walton Avenue in the Bronx is part of Region 1 in the DOE configuration of schools. The Region's focus for the school year 2004-2005 is growth in the writing process, so this research was timely as I incorporated the writing process into the study of mathematics. P.S. 64 is a school of 1100 students located in the Highbridge section of the Bronx. The school's ethnicity is 80.6% Hispanic, 17.9% African American and 1.5% Asian and others. Of the 80% Hispanic students, 37% have been identified as English Language Learners (ELL's). Of this 37%, 8.3% are recent immigrants to the United States, that is, they immigrated within the last three years. The English Language Learners receive exemptions in reading exams for three years. The Math tests are given to all students.

When the students entered the 3rd grade in September, all of them were just beginning to develop in the writing process. Sentences were rambling and somewhat incoherent. Certainly it could be said that writing did not come easily for any of them. This awareness was important to me as I engaged in this research about math writing.

## Review of the Literature

As I reflected daily on the students' work in Math, I turned to Marilyn Burns (1976,1993,1996), widely acknowledged as a leader in the on-going professional development of Math teachers. Burns shows that as students write about a problem solution they organize, clarify, and reflect on their ideas. Writing, she holds, provides students with concrete situations to see how many different ways there are to find solutions. Burns explains the variety of ways there are to use writing in math: keeping journals or logs, solving math problems, explaining mathematical ideas, and writing about learning processes. With this variety of assignments a teacher is able to gain insight into students' math experiences.

Joan Countryman (1992) discusses how students memorize examples, follow instructions, do their homework and take tests. But when someone asks them what they are doing and learning in math, the reply is often "we are on page 73". For those students it is an accurate answer. The lessons in the class generally follow in sequence, and page 73 is next. Math knowledge is pages to be covered, assignments to be completed, but maybe not about understanding.

Stephen Krulik and Jesse A. Rudnick (1993) write about the importance of developing thinking skills in young children. Their research is very

helpful as it gives specific suggestions for incorporating thinking activities into Everyday Math lessons. They point out that thinking/reasoning skills must be an integral part of mathematical development. Thinking has different levels: the lowest level is recall, which might include some of the basic arithmetic facts. Frequently, practice math pages rely on and encourage this level of thinking. The next level of thinking that Krulik and Rudnick identify is basic. This includes the understanding and recognition of mathematical concepts like addition, subtraction, and so on. Here again, pages in math books require a student to use this basic level of thinking. Their final two categories, critical thinking and creative thinking, are the most important, and can and should be enhanced and developed. The authors show how writing is one of the ways to develop critical and creative thinking.

Dominic Peressini and Norman Webb (1999) advocate that in order to support the NCTM (1989) standards of mathematical reasoning, new frame-works of responses must be developed that can facilitate analyses of students' responses. They propose that students do indeed have the ability to use mathematical language and communicate and solve a variety of problems.

As I continued to read and do research I realized that what I was doing was essential to my students' mathematical success, not perhaps the success required for weekly math quizzes or end-of-unit tests, but rather success in the ability to think creatively and find alternative solutions to problems. Along with this awareness, however, I realized that my students were not "natural" writers and that I would have to provide much encouragement into this venture. I was aware of the research that James Cummins (1981) did where he showed that it takes immigrant students considerably longer (five to seven years) to develop age-appropriate academic skills in English than to develop age-appropriate communicative skills in English (approximately two years). This informed research helped me to plan to combine lots of conversations and discussion with writing.

# The Study

## Data Collection Categories

I was catapulted into this action research when over a three week period in my classroom I observed conversations among the children during the math period. After engaging in a mini-lesson, on a specific concept or skill, the children work in small groups to practice the concept or skill. As I moved

about the room listening and answering questions, the conversations I heard were like this:

"What answer did you get?"

"I didn't get that."

"You must be wrong. I'm sure I did it right."

"Tell me the answer you got."

What concerned me most as I circulated among the children was that I never heard "How did you get your answer?" "I got my answer by doing this." "Did you do this?" The lack of critical thinking conversation was universal to all the students: I saw no distinction between good math students and struggling ones. All of them wanted only one thing: the right answer, and "Teacher" was the only one who knew the right answer. When I asked a Child, "How did you do this?" an answer rarely came. When asked, "How did you know what to do?" the response was usually, "I just know this is what you do." I began to wonder about my teaching and my students' learning.

## Data

Following this three week period, I decided to use a math journal as part of the development of critical thinking. Each of my 21 third graders prepared

and designed a Math Journal in which he or she would problem solve on a weekly basis. The problems chosen would be related to previous lessons, but the format and wording would be different from that which appears in the Everyday Math Book. I began with simple problems and advanced to two and three step problems.

The following is a sampling of the math problems. In each problem, the student was expected not only to solve the problem, but also to explain or tell why or how the solution was determined.

A. In which number does the **7** have the greatest value?

A. 1,725 C. 7,125

B. 5,271 D. 2,517

Tell why you chose your answer.

B. Which city has the least population?

A. St. Louis 351,565

B. Miami 365,127

C. Buffalo 310,548

D. Minneapolis 358,785

Explain why you chose your answer.

C. There were 64 teams at the beginning of basketball season. With 5 players starting on each team, how many starting players are there?

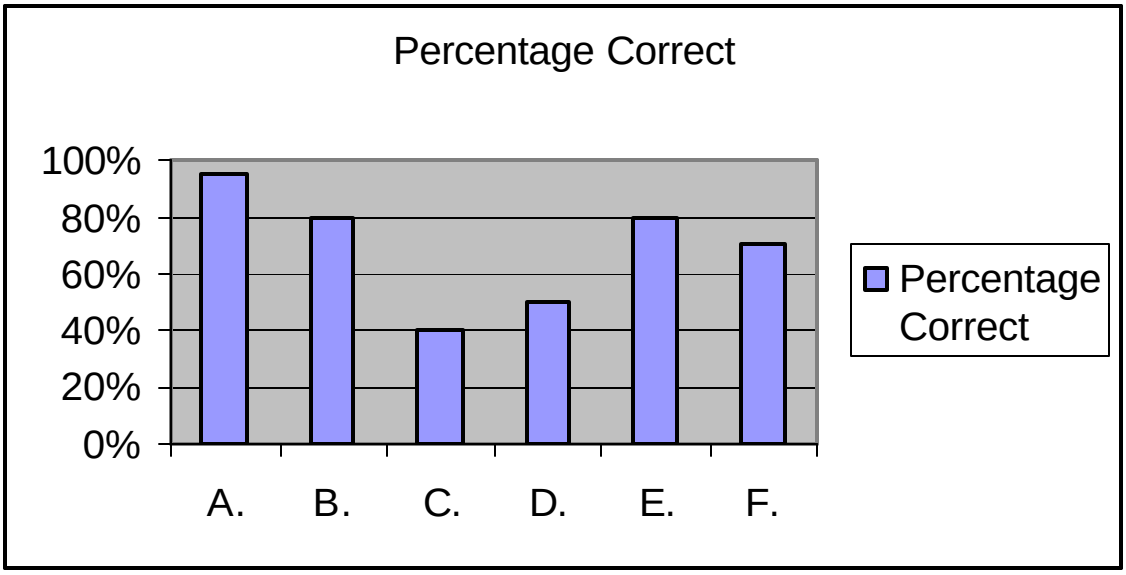
Use any strategy you know to solve this and then explain it.

D. There are 18 students in the Math Club. Today each student needs 12 bits to do a math project. How can we figure out the number of bits to give out? Tell what you did to solve this problem.

E. Thomas bought a package of baseball cards with 20 cards. He wants to share them equally with his 3 best friends and himself. What will he do? Explain and show your work.

F. Draw a rectangle shape. Make a long side of the rectangle 5 inches, and a short side of the rectangle 3 inches. What is the perimeter of this shape.

Tell how you found the answer.




My notes from these problem-solving activities reveal several things:

- In **Problem A** 95% of the students chose the correct answer, **C.** However, when they explained why they chose **C,** only 30% could tell that **C** was correct because **7** was in the thousands place. This indicated to me that there was not a clear understanding of place value, and while a place value chart was posted in the classroom, and all class discussions included place value terminology, students did not connect this problem to either the chart or conversations.
- In **Problem B** 80% of the students selected C - Buffalo. I believe the 20% of students who got it incorrect, confused the meaning of the word "least" with "most", since the answer they chose was the city with the most population. This made me very mindful of the English

Language Learners and their struggle with word meanings. However, Native English speakers also were among this 20%, so this indicated to me that there is a vocabulary deficit present.

- **Problem C** was challenging for many children. Initially, some children said it was too hard. In class we had not worked extensively on multiplying a two-digit number by a one digit number. I suggested to the children that they might draw a picture as a beginning strategy. As I observed the children working, several children multiplied 64 x 5 and got the answer. The larger number of children drew a picture or an array, but got confused in tallying the total number. This problem showed me how easily the children are stymied with unfamiliar wording, and do not immediately use known strategies. Only 40% of the class was able to solve the problem correctly.
- **Problem D** was presented after the previous one. It involved multiplying a two digit number by a two digit number. All of the students immediately drew a picture and tallied the tiles, but about 50% made computational errors in counting. This percentage did not concern me because I observed the students working on solutions: they were making connections to a previous problem and while the

final calculation was inaccurate, all of the students were involved in thinking.

- **Problem E** was universally solved by drawing pictures. Only 20% of the children got it wrong because they did not include Thomas as one of the sharers. Here again, I was observing the use of known strategies.
- **Problem F** was incorrectly solved by 30% of the children as they only added the two sides that had been mentioned in the problem. While these students can explain what perimeter is, when given a problem in which they must apply that knowledge, they do not. Should there be re-teaching about perimeter and area?

## Case Studies

In order to further illustrate the value and importance of writing in Math, I have chosen 3 students who are representative of the class population.

***Student 1***

Student 1 is a bright, alert, and hardworking student. He is a very competitive child and wants always to be the first one to come up with a solution. He gets personally offended if he cannot give the answer. At the

start of the year Student 1 wrote in his math autobiography: “When I’m about to do math I get scared and excited at the same time. I get scared because I’m not sure if it is correct and I’m excited because I like math.” This was written in September and this accurately describes how Student 1 approaches mathematical situations. The student enjoys number grids and skip counting by 3’s, 5’s, and 10’s. When asked to write whatever he knew about measurement, he wrote about the different units of measurement, but not when these units would be used. I observed that when answering a question he frequently left out an important element. As an illustration of this, I gave him a problem designed by Marilyn Burns, called ***The Firehouse Problem.*** These are the instructions:

> This is an unusual map of a town. On this map the lines represent streets, and the dots represent street corners. All the houses in the town are located at corners, and there’s at least one house on each corner.
>
> The town needs firehouses and the mayor has said that if a house catches on fire, fire trucks shouldn’t have to drive more than one street to get to it. It’s important that every house be on the same corner as a firehouse or only one street away.

> The problem is to figure out the fewest number of firehouses they can build and where to build them. Use cubes or other objects to mark where you think firehouses might go.

He looked at the diagram and said "This is too hard, I don't know what to do." I talked quietly with him and explained the directions. He settled down somewhat and began to work, and as I moved around, I observed that he approached another student and they began to work together. Watching Student 1 over the months, I realized that he indeed enjoys math and enjoys working computational problems and more important, wants them to be correct. I also realized that when confronted with unfamiliar wording or presentation, he loses confidence and needs support and encouragement to retrieve knowledge he possesses. His spirit of competitiveness supports him in computation, but not in critical thinking and reasoning. Student 1 is not considered an English Language Learner: English is the language spoken in the home.

***Student 2***

Student 2 is an average student who works hard at understanding math. Initially, adding and subtracting with re-grouping was a challenge, but

through steady practice, he has mastered it and is visibly excited with his success. In his math autobiography, Student 2 wrote "When I learn about math I get nervous because I think that I might get it wrong. I also think that times-tables are easy, so I like it." At the beginning of the year, this student became very serious at the start of the math period. He frequently requested individualized instructions and hardly ever raised his hand to contribute to a discussion. I have observed that this initial nervousness has abated and confidence has taken its place. In the math journal, Student 2 works slowly and very methodically. He uses known strategies: drawing pictures and diagrams, skip counting, tally marks, arrays, etc. He frequently is the last one to finish an assignment, but generally is able to explain why he solved the problem in a particular way. He is now an active contributor to class math solutions. Student 2 is not considered an English Language Learner: English is the language spoken in the home.

***Student 3***

Student 3 is repeating the third grade this year. His favorite subject is Literacy, both reading and writing. In a free-time choice, Student 3 always chooses to illustrate something he has written. Math is his nemesis. At the

beginning of the school year when math period would start, Student 3 would frequently refuse to stop writing/illustrating. His refrain was always "I don't like math." Engaging him in conversation I would tell him that he probably didn't like it because he felt unsure in math, and I promised to help him gain some confidence. Student 3 began his autobiography in this way: "Math is good for your brain; it makes you smart. Math, it is hard and I am going to do much better in math." At the end of Unit 1 when the children responded to an open-ended question about the material covered, Student 3 wrote "I think it was hard, because I had a hard time with it and my teacher helped me. Unit 1 is a little tough, but I still did it."

When telling me about measurement, Student 3 drew many pictures of shapes showing what the perimeter meant and writing that "measurement makes you smart." Student 3 has especially liked writing about math because he is able to use his favorite mediums: writing and drawing. Student 3 has a low level of recall and struggles constantly with addition and multiplication facts. This adds to his math frustration and perception of inadequacy. His problem solving strategies are not always computationally accurate, but his fear of the whole process has lessened, and his confidence and critical thinking abilities are growing. Student 3 is also not considered

an English Language Learner. However, Spanish is the language spoken in the home, and when an adult helps him with math, it is done in Spanish.

The results of the New York City CTB test are in. Student 1 and Student 2, who were presented in the Case Studies, scored a Level 3, meaning that they have met the standards for third grade. Student 3 scored a Level 1, meaning below grade standards. I made an appeal for this student, using the math journal that he had developed during the year since the work showed he is capable of performing at a high Level 2. The appeal was accepted and he will move to the fourth grade. As Student 3 and I were preparing his appeal portfolio, he told me that he wanted to write something, and these are his words: "I feel good about me because my math teacher made me be smart and now 2004 is all over. I learned a lot this year. I learned that when you could draw pictures it helps you find the answer, and I love to do math now that 2004 is done."

## Analysis

The data that I have gathered suggests that writing about math should be an integral part of an elementary math course of study, no matter what particular textbook publisher is used. In my case, where Everyday Math

(2000) is the chosen text, I learned that students come to frame their math knowledge in the familiar presentation and wording of the given text. Writing in Math requires students to analyze and identify patterns and relationships within problems and between problems. Students will compare problem-solving strategies and contexts of problem situations. In the case of Student 1, highly competitive and confident in "book format", thinking outside of this box was very challenging and at times baffling to him. The writing experience offered him an opportunity to build on the "known" and explore the "unknown" in a way reminiscent of those described by Krulik and Rudnick (1993) as critical and creative thinking. Student 2 was able to build confidence into his mastery of simple facts. Through writing he discovered how to use these facts in many different situations. I have observed that his initial nervousness has abated and confidence has taken its place. Student 3 was able to work in his comfort zone of drawing and writing which helped him get past his math block

While I observe real and tangible success for each student as they write and become "real" mathematicians, I also understand that English Language Learners have the added challenge of writing in their second language. In offering encouragement and support to these students, I accept expressions that would not be considered "correct English", and students are always

invited to read and discuss problem solutions. Throughout this study there was no judgment of the writing aspect. The only requirement was that it was readable. By posing problems to students, using everyday situations within their worlds, and asking them to find solutions, students are learning to integrate math into their everyday life experiences, and coming to "own" their knowledge.

The critical question going forward with this research is how to find the time to do math writing given the constraints of pacing calendars, etc. The average classroom teacher is very much controlled by Region and school expectations: everyone should be on the same page on the same day. Critical thinking and reasoning takes time: time to experiment, time to explain, time to work it out. And yes, there is no one who would argue that this is not the goal of all mathematics education, but as it plays out, time management becomes the big factor. Students who are neophytes to critical reasoning need time to explore: work with manipulatives, work with a partner, see which solution is better. This cannot be compressed into a routine fifteen or thirty minute segment of Math time.

How has this research affected my teaching? Teaching Math is no longer a "book" exercise, but hands-on, active engagement. Class discussions are about "How did you solve it?" or "Did anyone do it another

way?" No longer do I require students to complete every activity in the math book, but rather encourage students to make up problems and work together with a partner. Math is a time of exploration.

As I worked on this study, I devoted a math class every week to problem solving and critical thinking. I continue to do this. I am able to do this as my years of teaching experience have informed me about how to choose the key principles in math that must be taught. New and beginning teachers would need guidance and coaching to help them implement writing in math. And to answer the question with which I started: ***How does student writing about Math connect to the assessment of students' mathematical understanding?***

The action research that I did supports an understanding that writing is integral to assessment. Student 3, while not being successful on a standardized test, showed understanding and comprehension when able to write about his math activity. There can be no complete assessment without writing. Other students whom I perceived to understand a key concept, when confronted with a reasoning problem, experienced difficulty in finding a solution. These students were unable to integrate the knowledge they had and apply it to a new situation.

## New Questions for Research

My study was limited to my classroom which is part of a low-achieving school in the Bronx. Many of the students are English Language Learners. For purposes of validation, further research should include students who attend academically successful schools. Would there be the same findings among the students in such a setting? And then further action research would address:

- How does writing in math impact on standardized Math test scores? Does it help to raise scores?
- What is the impact of learning a second language on the development of critical reasoning and thinking skills?
- How would daily writing in math impact mathematics education in junior and senior high school, and thus allow the United States to once again become a leader in critical thinking and analysis?
- Should mathematics education have a more distant relationship with textbook materials?

## Policy Recommendations

- My study suggests that writing in math increases students understanding and comprehension of key math concepts. It gives students an opportunity to organize, clarify and think about what they have learned.
- At the classroom level one day each week should be dedicated to the development of critical thinking skills through the use of journaling, problem solving and math explorations.
- At the district/region level, pacing calendars should be developed to allow for flexibility. As teachers continually work towards differentiating instruction, they must be given freedom to design lessons that are "outside the box" of pacing.
- True assessments have two components: standardized tests and portfolio pieces that demonstrate mastery of knowledge and understanding. In evaluating students, one must always look through the lens of both of these.

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